

# Representing Temporal Operators with Dependent Event Types

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Event semantics were first proposed and studied by Davidson to express the occurrence of events in order to find suitable semantics for describing both actions and adverbial modifications [2], and further studied and developed by many in the fields of logic, philosophy and computer science [10, 8, 3, 9]. Later works showed that dependent types theories were suitable for formalising event semantics in the form of dependent event types, which allow for selection restriction through semantic roles such as the agent of an event [5].

In this work, we propose a further refinement of dependent event types by extending a type system with dependent event types by arbitrary semantic roles. We show that in the case of Church’s simple type theory with dependent event types, this extension by arbitrary semantic roles is conservative. We focus on the semantic role of time via the introduction of a timeline type `Time` and subtypes of events parameterised by a timestamp, and use this to formulate traditional temporal operators and explore their properties in this system.

**Dependent Event Types.** In Davidsonian event semantics, there exists a type `Evt` of all events, and each event contains information associated with that event’s semantic roles. When working in with Montague grammar, we can give meaning to sentences by interpreting them as types and using predicates to restrict their arguments. For example, we can interpret the sentence “Claire eats an apple” with agent ‘Claire’ and patient ‘apple’ as the type

$$\exists(e : \text{Evt}).\text{agent}(e, \text{Claire}) \wedge \text{patient}(e, \text{apple}) \wedge \text{eats}(e).$$

The development of dependent event types allows one to consider subtypes of `Evt` parameterised by an event’s semantic roles, such as the agent of an event. By extending the type system with dependent event types and new types for each semantic role<sup>1</sup>, we can instead consider subtypes of `Evt` parameterised by semantic roles. For example, to express the sentence “Claire eats an apple” this way, we need the following rules

$$\frac{\Gamma \vdash a : \text{Agent}}{\Gamma \vdash \text{Evt}_A(a) \text{ type}} \quad \frac{\Gamma \vdash a : \text{Agent}}{\Gamma \vdash \text{Evt}_P(p) \text{ type}} \quad \frac{\Gamma \vdash a : \text{Agent} \quad \Gamma \vdash p : \text{Patient}}{\Gamma \vdash \text{Evt}_{AP}(a, p) \text{ type}}$$

and subsumptive subtyping relations

$$\frac{\Gamma \vdash a : \text{Agent}}{\Gamma \vdash \text{Evt}_A(a) \leq \text{Evt}} \quad \frac{\Gamma \vdash p : \text{Patient}}{\Gamma \vdash \text{Evt}_P(p) \leq \text{Evt}}$$

$$\frac{\Gamma \vdash a : \text{Agent} \quad \Gamma \vdash p : \text{Patient}}{\Gamma \vdash \text{Evt}_{AP}(a, p) \leq \text{Evt}_A(a)} \quad \frac{\Gamma \vdash a : \text{Agent} \quad \Gamma \vdash p : \text{Patient}}{\Gamma \vdash \text{Evt}_{AP}(a, p) \leq \text{Evt}_P(p)}.$$

In this framework, we are able to express this sentence as

$$\exists(e : \text{Evt}_{AP}(\text{Claire}, \text{apple})).\text{eats}(e)$$

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<sup>1</sup>In the case of the given example, the semantic roles of agent and patient.

where  $\text{eats} : \text{Evt} \rightarrow \mathbf{t}$ , which is well-typed due to our use of subtyping.

Prior work on dependent event types has shown that extending Church’s simple type theory with dependent event types is a conservative extension [6]. We extend these results to show that further extending this system with arbitrary semantic roles and subtypes of  $\text{Evt}$  parameterised by these semantic roles is also a conservative extension.

**Theorem 1** (Conservativity). *Let  $C$  be Church’s simple type theory. Fix  $S$  a collection of semantic roles, and let  $C_E[S]$  be  $C$  extended by dependent event types with subtypes parameterised by semantic roles in  $S$ . Then  $C_E[S]$  is a conservative extension of  $C$ .*

Of particular interest, this means that extending dependent event types with time is conservative. We also extend the subtyping of  $\text{Evt}$  to allow for one to parameterise the time that an event occurs ( $\text{Evt}_T$ ) and also for an interval in which an event occurs ( $\text{Evt}_{TT}$ ).

**Temporal operators.** One of the original motivations for this work was to further study a modal type theory from a rules-first approach, where most prior work on modal type theories have been from a model-first approach. In particular, there was interest in extending dependent event types with the semantic role of time to enable the description and use of temporal operators.

There are typically two perspectives that are used in the study of temporal logic: those concerned with the analysis of computer software are likely to view temporal operators as a kind of function that takes as input a given moment in time and checks for truth of a given proposition at that moment of time; whereas those concerned with the description of natural language are likely to take the view that each statement has a ‘speaking time’, an inherent moment in time which it is in reference to, and can only be evaluated in comparison to that speaking time. Our work starts with the latter as a basis, and extends it to the former.

We can describe the traditional  $\Box$  and  $\Diamond$  operators

$$\Diamond A \stackrel{\text{def}}{=} \exists(t : \text{Time}).(\text{now} \preceq t \wedge A(t))$$

$$\Box A \stackrel{\text{def}}{=} \forall(t : \text{Time}).(\text{now} \preceq t \rightarrow A(t))$$

and use these to encode and represent simple sentences such as “John will talk” as  $\Diamond A$ , where  $A(t) \stackrel{\text{def}}{=} \exists(e : \text{Evt}_{AT}(\text{John}, t)).\text{talk}(e)$ . However, these are rather restrictive in their use due to their fixed speaking time. While these can be used to express the sentence “there will be flying cars in the future”, these fail to carry the importance of what point in time this sentence was instantiated or spoken. If we read this sentence in the year 2024, it carries different information and different meaning when it was spoken in the year 1985 versus the year 2023. We can adapt the above temporal operators to allow for variable speaking time.

$$\Diamond A(\text{ref}) \stackrel{\text{def}}{=} \exists(t : \text{Time}).(\text{ref} \preceq t \wedge A(t))$$

$$\Box A(\text{ref}) \stackrel{\text{def}}{=} \forall(t : \text{Time}).(\text{ref} \preceq t \rightarrow A(t))$$

Taking this approach also changes their type signature from  $(\text{Time} \rightarrow \mathbf{t}) \rightarrow \mathbf{t}$  to  $(\text{Time} \rightarrow \mathbf{t}) \rightarrow (\text{Time} \rightarrow \mathbf{t})$ , which allows us to nest multiple temporal operators together, e.g.  $\Box \Diamond A$ . This allows us to have the important distinction of different speaking times. For example, one may consider the sentence “John will eventually always have grey hair” encoded as  $\Diamond(\Box A)$  *now*, where  $A$  is the sentence “John has grey hair.”

**Conclusion.** This work is currently ongoing, and we plan to extend our results from extending Church’s simple type theory to extending modern type theories such as Martin-Löf’s type theory. For Church’s simple type theory, Montague grammar is a well-studied approach to natural language semantics which allows one to express the categorisation of objects by checking that they satisfy ‘checking’ propositions, but this can lead to syntactically correct but categorically incorrect sentences [11]. Different approaches prevent this, such as the use of subtyping to express the relationships between categories [1, 4], or by using a modern type theory to ensure that only categorically correct sentence are well-typed [12]. For example, “John is speaking” could be interpreted as

$$\exists(e : \text{Evt}_A(\text{John})).\text{speaks}(e)$$

where  $\text{speaks} : \text{Evt} \rightarrow \mathbf{t}$  and the inhabitation of this type expresses the truth of this sentence. On the other hand, while one may be able to form an expression for a sentence such as “the table is talking,” it could not be a well-typed sentence without further providing the sentence further context, such as  $\text{Table} \leq \text{Human}$ .

However, working with Church’s simple type theory allows for the use of subsumptive subtyping when describing subtypes of  $\text{Evt}$ , whereas working to extend modern type theories which allow for dependent types, polymorphism, and other more complex expressions requires a more nuanced approach through coercive subtyping [7].

Dependent event types extended with the semantic role of time bears some resemblance to functional reactive programming. However, this is outside the scope of this work.

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